Electric Charges and fields

- Electric and magnetic forces determine the properties of atoms, molecules and bulk matter.
- From simple experiments on frictional electricity, one can infer that there are two types of charges in nature; and that like charges repel and unlike charges attract. By convention, the charge on a glass rod rubbed with silk is positive; that on a plastic rod rubbed with fur is then negative.
- Conductors allow movement of electric charge through them, insulators do not. In metals, the mobile charges are electrons; in electrolytes both positive and negative ions are mobile.
- Electric charge has three basic properties: quantisation, additivity and conservation.

  Quantisation of electric charge means that total charge (q) of a body is always an integral multiple of a basic quantum of charge (e) i.e., $q = n e$, where $n = 0, \pm1, \pm2, \pm3, ...$ Proton and electron have charges $+e$, $–e$, respectively. For macroscopic charges for which n is a very large number, quantisation of charge can be ignored. Additivity of electric charges means that the total charge of a system is the algebraic sum (i.e., the sum taking into account proper signs) of all individual charges in the system.

- Conservation of electric charges means that the total charge of an isolated system remains unchanged with time. This means that when bodies are charged through friction, there is a transfer of electric charge from one body to another, but no creation or destruction of charge.
- Coulomb's Law: The mutual electrostatic force between two pointcharges q1 and q2 is proportional to the product q1q2 and inverselyproportional to the square of the distance r21 separating them.

  Mathematically,

  F21 = force on q2 due to q1 = $k(q1q1)/r_{21}^{2}$

  K = (1/4π ε0)

  In SI units, the unit of charge is coulomb. The experimental value of the constant ε0 is ε0 = 8.854×10^{-12} C2 N^{-1} m^{-2}

  The approximate value of k is k = 9×10^{9} N m^{2} C^{-2}
- The ratio of electric force and gravitational force between a proton and an electron is

  $ke^{2}/Gm_{e}m_{p} = 24 * 10^{39}$

- Superposition Principle: The principle is based on the property that the forces with which two charges attract or repel each other are not affected by the presence of a third (or more) additional charge(s). For an assembly of charges q1, q2, q3, ..., the force on any charge, say q1, is the vector sum of the force on q1 due to q2, the force on q1 due to q3, and so on. For each pair, the force is given by the Coulomb's law for two charges stated earlier.
- The electric field E at a point due to a charge configuration is theforce on a small positive test charge q placed at the point divided bythe magnitude of the charge. Electric field due to a point charge q hasa magnitude $|q|/4\pi\varepsilon 0r^2$; it is radially outwards from q, if q is positiveand radially inwards if q is negative. Like Coulomb force, electric fieldalso satisfies superposition principle.
- An electric field line is a curve drawn in such a way that the tangentat each point on the curve gives the direction of electric field at thatpoint. The relative closeness of field lines indicates the relative strengthof electric field at different points; they crowd near each other in regionsof strong electric field and are far apart where the electric field isweak. In regions of constant electric field, the field lines are uniformlyspaced parallel straight lines.
- Some of the important properties of field lines are:

  (i) Field lines arecontinuous curves without any breaks.

(ii) Two field lines cannot crosseach other.

(iii) Electrostatic field lines start at positive charges andend at negative charges —they cannot form closed loops.

- An electric dipole is a pair of equal and opposite charges q and –qseparated by some distance 2a. Its dipole moment vector p hasmagnitude 2qa and is in the direction of the dipole axis from –q to q.
- Field of an electric dipole in its equatorial plane (i.e., the planeperpendicular to its axis and passing through its centre) at a distancer from the centre:

  $E = (-p/4\pi\varepsilon 0)[1/(a^2 + r^2)^{3/2}]$

  $= -p/(4\pi\ \varepsilon 0r^3)$ for r>>a

  Dipole electric field on the axis at a distance r from the centre

  $E = (2pr)/(\ 4\pi\ \varepsilon 0(r^2 - a^2)^2)$

  $= 2p(4\pi\ \varepsilon 0r^3)$ for r>>a

The $1/r^3$ dependence of dipole electric fields should be noted in contrast to the $1/r^2$ dependence of electric field due to a point charge.

- In a uniform electric field **E**, a dipole experiences a torque τ given by

  $\tau = \mathbf{p} \times \mathbf{E}$

  but experiences no net force.

- The flux Δϕ of electric field **E** through a small area element Δ**S** is given by

  $\Delta\phi = \mathbf{E}.\Delta\mathbf{S}$

  The vector area element Δ**S** is

  $\Delta\mathbf{S} = \Delta S\, \hat{\mathbf{n}}$

  where ΔS is the magnitude of the area element and $\hat{\mathbf{n}}$ is normal to the area element, which can be considered planar for sufficiently small ΔS.

- Gauss's law: The flux of electric field through any closed surface S is$1/\varepsilon_0$ times the total charge enclosed by S. The law is especially usefulin determining electric field E, when the source distribution has simplesymmetry:

  (i) Thin infinitely long straight wire of uniform linear charge density λ

  $E = \lambda / 2\pi \varepsilon_0 r$

  where r is the perpendicular distance of the point from the wire and $\hat{\mathbf{n}}$ is the radial unit vector in the plane normal to the wire passing through the point.

  (ii) Infinite thin plane sheet of uniform surface charge density σ

  $E = \sigma/2\varepsilon_0$

  where $\hat{\mathbf{n}}$ is a unit vector normal to the plane, outward on either side.

  (iii) Thin spherical shell of uniform surface charge density σ

  $E = q/4\pi\varepsilon_0 r \quad (r >= R)$

  E where r is the distance of the point from the centre of the shell and R the radius of the shell. q is the total charge of the shell: $q = 4\pi R^2\sigma$. The electric field outside the shell is as though the total charge is concentrated at the centre. The same result is true for a solid sphere of uniform volume charge density. The field is zero at all points inside the shell = 0 $\quad (r < R)$

Sample Examples

- If 10^9 electrons move out of a body to another bodyevery second, how much time is required to get a total charge of 1 Con the other body?

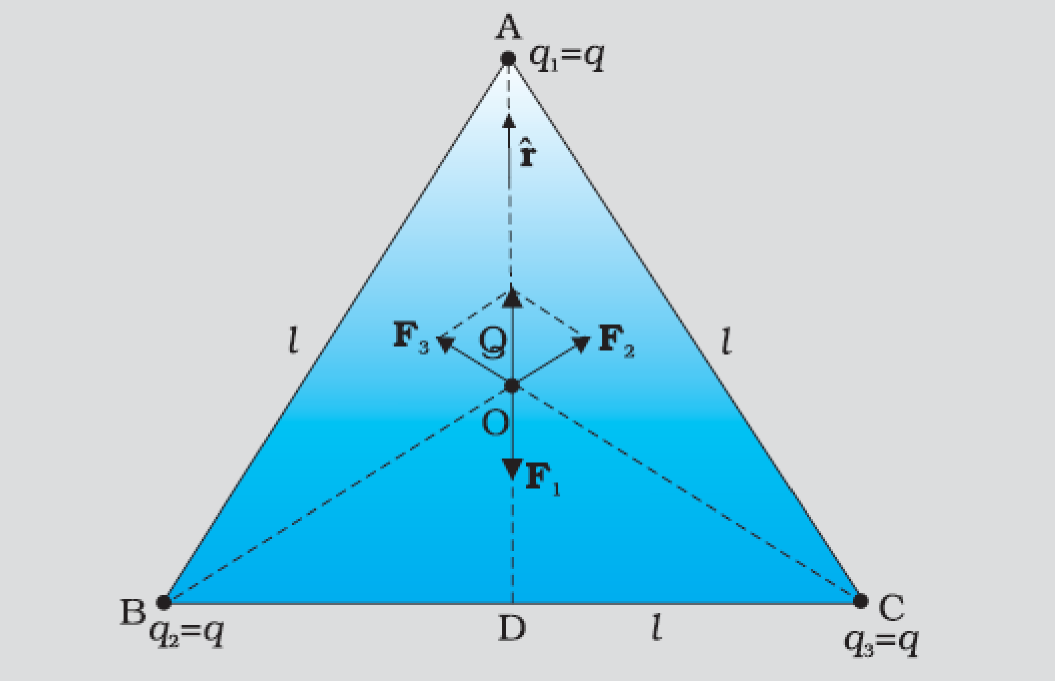
  Solution

  In one second 109 electrons move out of the body. Thereforethe charge given out in one second is $1.6 \times 10^{-19} \times 10^9$ C = 1.6×10^{-10} C.The time required to accumulate a charge of 1 C can then be estimatedto be 1 C ÷ (1.6×10^{-10} C/s) = 6.25×10^9 s = $6.25 \times 10^9 \div (365 \times 24 \times 3600)$ years = 198 years. Thus to collect a charge of one coulomb,
  from a body from which 10^9 electrons move out every second, we willneed approximately 200 years. One coulomb is, therefore, a very largeunit for many practical purposes.

  It is, however also important to know what is roughly the number ofelectrons contained in a piece of one cubic centimetre of a material.

  A cubic piece of copper of side 1 cm contains about 2.5×10^{24}electrons.

- Consider three charges q1, q2, q3 each equal to q at thevertices of an equilateral triangle of side l. What is the force on acharge Q (with the same sign as q) placed at the centroid of the triangle?



In the given equilateral triangle ABC of sides of length l, if we draw a perpendicular AD to the side BC,

AD = AC cos 30º = (√3/2) l and the distance AO of the centroid O from A is (2/3) AD = (1/√ 3) l. By symmetry AO = BO = CO.

Force **F1** on Q due to charge q at A = $3Qq/42\pi\varepsilon 0l^2$ AO

Force **F2** on Q due to charge q at A = $3Qq/42\pi\varepsilon 0l^2$ BO

Force **F3** on Q due to charge q at A = $3Qq/42\pi\varepsilon 0l^2$ CO

The resultant of forces **F2** and **F3** is = $3Qq/42\pi\varepsilon 0l^2$ along AO by the parallelogram law.

Therefore, the total force on Q = $3Qq/42\pi\varepsilon 0l^2(r - r) = 0$,

where **r**ˆ is the unit vector along OA. It is clear also by symmetry that the three forces will sum to zero.